

Curved and diffuse interface effects on the nuclear surface tension

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Abstract

We redefine the surface tension coefficient for a nuclear Fermi-liquid drop with a finite diffuse layer. Following Gibbs-Tolman concept, we introduce the equimolar radius R_e of sharp surface droplet at which the surface tension is applied and the radius of tension surface R_s which provides the minimum of the surface tension coefficient σ . This procedure allows us to derive both the surface tension and the corresponding curvature correction (Tolman length) correctly for the curved and diffuse interface. We point out that the curvature correction depends significantly on the finite diffuse interface. This fact is missed in traditional nuclear considerations of curvature correction to the surface tension. We show that Tolman's length ξ is negative for nuclear Fermi-liquid drop. The value of the Tolman length is only slightly sensitive to the Skyrme force parametrization and equals $\xi = -0.36$ fm.

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I. INTRODUCTION

The binding energy of saturated many-particle systems like a nucleus exhibits the hierarchy of expansion in powers of mass number $A^{1/3}$ which is associated with well-established experimental features and plays an important role in the understanding of macroscopic properties of nuclei [1]. In a simplest case, the surface energy E_S of such systems is given by the term of order $A^{2/3}$ in this hierarchy. The structure of the surface energy E_S and the corresponding surface tension coefficient σ depend on the interparticle interaction and the surface conditions. Moreover, the nucleus is a two component, charged system with a finite diffuse layer. This fact specifies a number of various peculiarities of the nuclear surface energy E_S : dependency on the density profile function, contribution to the surface symmetry energy, connection to the nuclear incompressibility, etc. The additional refinements of E_S appear due to the quantum effects arising from the smallness of nucleus. In particular, the curved interface creates the curvature correction to E_S of order $A^{1/3}$ and can play the appreciable role in small nuclei.

The curvature correction to the planar tension coefficient and the corresponding Tolman length [2] can be estimated phenomenologically using the polynomial, in powers of $A^{1/3}$, expansion of mass formula [3–6]. The importance of curvature term for the evaluation of nuclear masses and fission barriers and the interplay between different terms of the $A^{1/3}$ -expansion due to the value of the radius was shown in [7]. However the influence of the curved interface on the properties of small quantum systems is still poorly studied because of the finite diffuse layer where particle density drops down to the zero value. The presence of the finite diffuse layer in a small drop creates two, at least, questions: (i) What is the actual radius of a drop? (ii) What is the physical surface where the surface tension is applied? Since the presence of the diffuse layer, different definitions for the size of the drop are possible [8] which all give the value of the radius of spherical drop located within the diffuse layer.

Note that a small indefiniteness in a nuclear radius of the order of $\Delta R \approx 0.5$ fm, i.e., within the diffuse layer, leads to a shift of surface energy of order $\Delta E_S \approx 10^2$ MeV (for ^{208}Pb [1]) which exceeds significantly both the shell [9] and the curvature [4] corrections to the nuclear binding energy, see also ref. [7]. We point out also even though the width of diffuse layer is much less than the range of approximate uniformity of the particle density, one still

needs the strict definition of the drop size because of the following reason. In contrast to the planar geometry, the area S for the spherical (curved) surface will depend on the choice of drop radius and this will affect the value of the surface tension σ derived from the surface energy.

Gibbs was the first who addressed the problem of the correct definition of the radius and the surface of tension to a small drop with a diffuse interface [11]. After him, Tolman drew attention [2] that two different radii have to be introduced in this case: the equimolar radius R_e , which gives the actual size of the corresponding sharp-surface droplet, and the radius of tension R_s , which derives, in particular, the capillary pressure, see below in sect. II. Following Tolman, see also ref. [10], the surface tension $\sigma_e \equiv \sigma(R_e)$ approaches the planar limit σ_∞ as

$$\sigma(R_e) = \sigma_\infty \left(1 - \frac{2\xi}{R_e} + \mathcal{O}(R_e^{-2}) \right), \quad (1)$$

where ξ is the Tolman's length [2]. At the same time the capillary pressure P_{capil} , which is generated by the curved surface and provides the equilibrium condition for the well-defined radius R_e , is derived by the radius of tension R_s [11]

$$P_{\text{capil}} = \frac{2\sigma}{R_s}. \quad (2)$$

In general, the presence of the curved interface affects both the bulk and the surface properties. The curvature correction $\Delta\sigma_{\text{curv}} = -2\sigma_\infty\xi/R_e \sim A^{-1/3}$ is usually negligible in heavy nuclei. However, this correction can be important in some nuclear processes. That the yield of fragments at the nuclear multifragmentation or the probability of clusterization of nuclei from the freeze-out volume in heavy ion collisions are derived by the statistical weight W of the radius fluctuations [12]

$$W \propto e^{-\text{const } \sigma R^2/T}.$$

In both above mentioned processes, the small nuclei occur necessarily and the exponential dependence of statistical weight W on the surface tension σ should cause the sensitivity of both processes to the curvature correction $\Delta\sigma_{\text{curv}}$. The curvature correction can also play appreciable role for the nuclear fission near the scission point and for the nuclear fusion in the neck region. This aspect of large nuclear deformations was not studied yet.

In nuclear physics, the curvature correction to the surface tension was intensively investigated phenomenologically [3, 4, 6] as well as within the quantum approaches [13]. Using the

Thomas-Fermi approximation, the dependence on curvature of the nuclear surface energy was studied in ref. [14] and the various terms of the droplet model were derived from the Skyrme interaction in ref. [15]. In ref. [16], a general procedure restricted by the Skyrme-type functional and the terms of the order \hbar^2 was applied for calculation of the curvature energy and in ref. [17] the curvature-energy was studied by use of semiclassical mean-field approaches with including higher-order terms. In ref. [18] a closed expression for the nuclear curvature energy and its expansion in series of volume terms and surface moments was deduced in a soluble model. For the Fermi gas model in an external Woods-Saxon potential the curvature energy was calculated in ref. [19]. In ref. [7], the nuclear liquid-drop model containing the first and/or the second order curvature terms was revised and it was reproduced with a reasonable precision the nuclear binding energies and the fission-barriers based on large amount of up-to-date experimental data. A special care was given to the kinetic energy operator and the bulk density oscillations within the quantum-mechanical approach [13]. It was shown that, in contrast to the semiclassical approaches, to obtain the reasonable value for the curvature energy the particle and energy densities should be averaged in a special way.

In present paper, we suggest the microscopic analysis of the curvature correction to the surface tension of a small drop with a finite diffuse layer. We follow the ideology of the extended Thomas-Fermi approximation (ETFA) with effective Skyrme-like forces combining the ETFA and the direct variational method. In our consideration, the proton and neutron densities $\rho_p(\mathbf{r})$ and $\rho_n(\mathbf{r})$ are generated by the diffuse-layer profile functions which are eliminated by the requirement that the energy of the nucleus be stationary with respect to variations of these profiles. In order to formulate proper definition for the drop radius, we will use the concept of dividing surface, originally introduced by Gibbs [11]. Following Gibbs, we will introduce the superficial (surface) density as the difference (per unit area of dividing surface) between actual number of particles A and the number of particles A_γ which drop would contain if the particle density retained uniform.

This paper is organized as follows. In sect. II we give the thermodynamical derivation of the surface tension for a finite system. The Tolman length is derived in sect. III. Numerical results and conclusions are summarized in sects. IV and VI. The connection of Gibbs-Tolman approach to the droplet model is given in sect. V.

II. EQUIMOLAR SURFACE

We will calculate the dependence of surface tension coefficient on the position of the dividing surface in a small Fermi-liquid drop with a finite diffuse layer similarly to procedure described in refs. [10, 20]. The goal of calculations is to determine the position of the equimolar surface, the dependence of surface tension on the bulk density and the sensitivity of the curvature correction (Tolman length ξ) to the parametrization of the effective nuclear forces.

We consider the uncharged symmetric ($N = Z$) droplet having number of particles $A = N + Z$, chemical potential λ and free energy F . Note that the thermodynamical consideration is most adequate here because of the finite diffuse interface in a cold nucleus is similar to the vapor environment in a classical liquid drop. In order to formulate proper definition for the drop radius, we will use the concept of dividing surface of radius R , originally introduced by Gibbs [11]. Following refs. [10, 11], we introduce the formal (arbitrary but close to the interface) dividing surface of radius R , the corresponding volume $\mathcal{V} = 4\pi R^3/3$ and the surface area $\mathcal{S} = 4\pi R^2$. The droplet free energy F will be then split between volume, $F_{\mathcal{V}}$, and surface, $F_{\mathcal{S}}$, parts

$$F = F_{\mathcal{V}} + F_{\mathcal{S}} , \quad (3)$$

where

$$F_{\mathcal{V}} = (-P + \lambda \varrho_{\mathcal{V}}) \mathcal{V} , \quad F_{\mathcal{S}} = (\sigma + \lambda \varrho_{\mathcal{S}}) \mathcal{S} . \quad (4)$$

Here, $P = P(\lambda)$ is the pressure of nuclear matter achieved at some volume particle density $\varrho_{\mathcal{V}} = A_{\mathcal{V}}/\mathcal{V}$ and $\varrho_{\mathcal{S}} = A_{\mathcal{S}}/\mathcal{S}$ is the surface density, where $A_{\mathcal{V}}$ and $A_{\mathcal{S}}$ are the volume and the surface particle number, respectively. The actual particle number is given by

$$A = A_{\mathcal{V}} + A_{\mathcal{S}} = \varrho_{\mathcal{V}} \mathcal{V} + \varrho_{\mathcal{S}} \mathcal{S} . \quad (5)$$

The use of eqs. (3) – (5) gives the following relation for the surface tension

$$\sigma = \frac{F - \lambda A}{\mathcal{S}} + \frac{P \mathcal{V}}{\mathcal{S}} = \frac{\Omega - \Omega_{\mathcal{V}}}{\mathcal{S}} , \quad (6)$$

where symbol $\Omega = F - \lambda A$ stands for the grand potential and $\Omega_{\mathcal{V}} = -P \mathcal{V}$. To reveal a R -dependence of surface tension σ , it is convenient to introduce the grand potential per particle $\omega = F/A - \lambda$ for the actual droplet and $\omega_{\mathcal{V}} = F_{\mathcal{V}}/A_{\mathcal{V}} - \lambda = -P/\varrho_{\mathcal{V}}$ for the volume

part. Then the surface tension is written as

$$\sigma [R] = \frac{\omega A}{4\pi R^2} - \frac{1}{3}\omega_{\nu}\varrho_{\nu}R . \quad (7)$$

Here, the square brackets denote a dependence of the observable F , λ , P etc. on the dividing surface radius R which is different than the dependence on the physical size of a droplet [20]. Using eq. (6), the capillary pressure P reads

$$P = 3\frac{\sigma [R]}{R} - 3\frac{F - \lambda A}{4\pi R^3} . \quad (8)$$

Taking the derivative from eq. (8) with respect to the formal dividing radius R and using the fact that the observable F , λ and P are R -independent (changing dividing radius R we keep the particle density invariable), one can rewrite eq. (8) as

$$P = 2\frac{\sigma [R]}{R} + \frac{\partial}{\partial R}\sigma [R] , \quad (9)$$

which is the generalized Laplace equation.

The choice of the dividing radius R is arbitrary, the only condition is to keep the same chemical potential λ . So, the formal value of surface density ϱ_S can be positive or negative depending on R . From eq. (5) one finds

$$\varrho_S[R] = \frac{A}{4\pi R^2} - \frac{1}{3}\varrho_{\nu}R . \quad (10)$$

We have performed the numerical calculations using Skyrme type of the effective nucleon-nucleon interaction. The energy and the chemical potential for actual droplet have been calculated using direct variational method within the extended Thomas-Fermi approximation [21]. Assuming the leptodermous condition, the total energy takes the following form of A , X -expansion

$$F/A \equiv e_A = e_0(A) + e_1(A)X + e_2(A)X^2 , \quad (11)$$

where X is the isotopic asymmetry parameter $X = (N - Z)/A$ and

$$e_i(A) = c_{i,0} + c_{i,1}A^{-1/3} + c_{i,2}A^{-2/3} . \quad (12)$$

An explicit form of coefficients $c_{i,j}$ for the Skyrme forces is given in ref. [21]. Using the trial profile function for the neutron $\rho_n(r)$ and proton $\rho_p(r)$ densities and performing the direct variational procedure, we can evaluate the equilibrium particle densities $\rho_{\pm}(r) =$

$\rho_n(r) \pm \rho_p(r)$, equilibrium bulk density $\rho_{\pm,0} = \lim_{r \rightarrow 0} \rho_{\pm}(r)$, total free energy per particle F/A and chemical potentials λ_n and λ_p , see ref. [21] for details. From now on, we will consider symmetric and uncharged nuclei with $\lambda_n = \lambda_p = \lambda$ and $X = 0$.

The volume part of free energy F_V/A_V is associated with coefficient $c_{0,0}$ of ref. [21]

$$F_V/A_V = c_{0,0}, \quad c_{0,0} = \frac{\hbar^2}{2m} \alpha \rho_{+,0}^{2/3} + \frac{3t_0}{8} \rho_{+,0} + \frac{t_3}{16} \rho_{+,0}^{\nu+1} + \frac{\alpha}{16} [3t_1 + t_2(5 + 4x_2)] \rho_{+,0}^{5/3}, \quad (13)$$

where $\alpha = (3/5) (3\pi^2/2)^{2/3}$ and t_i , x_2 and ν are the Skyrme force parameters. Using the evaluated chemical potential λ , we fix the particle density $\varrho_V = \varrho_V(\lambda)$ from the condition

$$\left. \frac{\partial F_V}{\partial A_V} \right|_V = \left. \frac{\partial}{\partial \rho_{+,0}} (\rho_{+,0} c_{0,0}) \right|_{\rho_{+,0} = \varrho_V} = \lambda. \quad (14)$$

For an arbitrary dividing radius R we evaluate then the volume particle number $A_V = 4\pi R^3 \varrho_V/3$ and the volume part of free energy F_V/A_V . Finally, evaluating the surface parts $A_S = A - A_V$ and $F_S = F - F_V$, we obtain the surface tension coefficient $\sigma[R]$ for an arbitrary radius R of dividing surface.

The dependence of the surface tension $\sigma[R]$ on the location of the dividing surface for $A = 208$ is shown in FIG. 1. As seen from FIG. 1, function $\sigma[R]$ has minimum at radius $R = R_s$ (radius of surface of tension [10]) which usually does not coincide with the equimolar radius R_e . The radius R_s denotes the location within the interface. Note that for $R = R_s$ the capillary pressure of eq. (9) satisfies the classical Laplace relation

$$P = 2 \left. \frac{\sigma[R]}{R} \right|_{R=R_s}. \quad (15)$$

III. SURFACE TENSION AND TOLMAN LENGTH

In FIG. 2 we present the calculation of the surface particle density $\varrho_S[R]$. Note that, in general, the surface free energy F_S includes both contributions from the surface tension σ itself and from the bulk binding energy of A_S particles within the surface layer. The equimolar surface and the actual physical radius R_e of the droplet is derived by the condition $\varrho_S[R_e] = 0$ [2, 10, 20], i.e., the contribution from the bulk binding energy should be excluded from the surface free energy F_S . The corresponding radius is marked in FIG. 2. Equimolar dividing radius R_e defines the physical size of the sharp surface droplet and the surface at which the surface tension is applied.

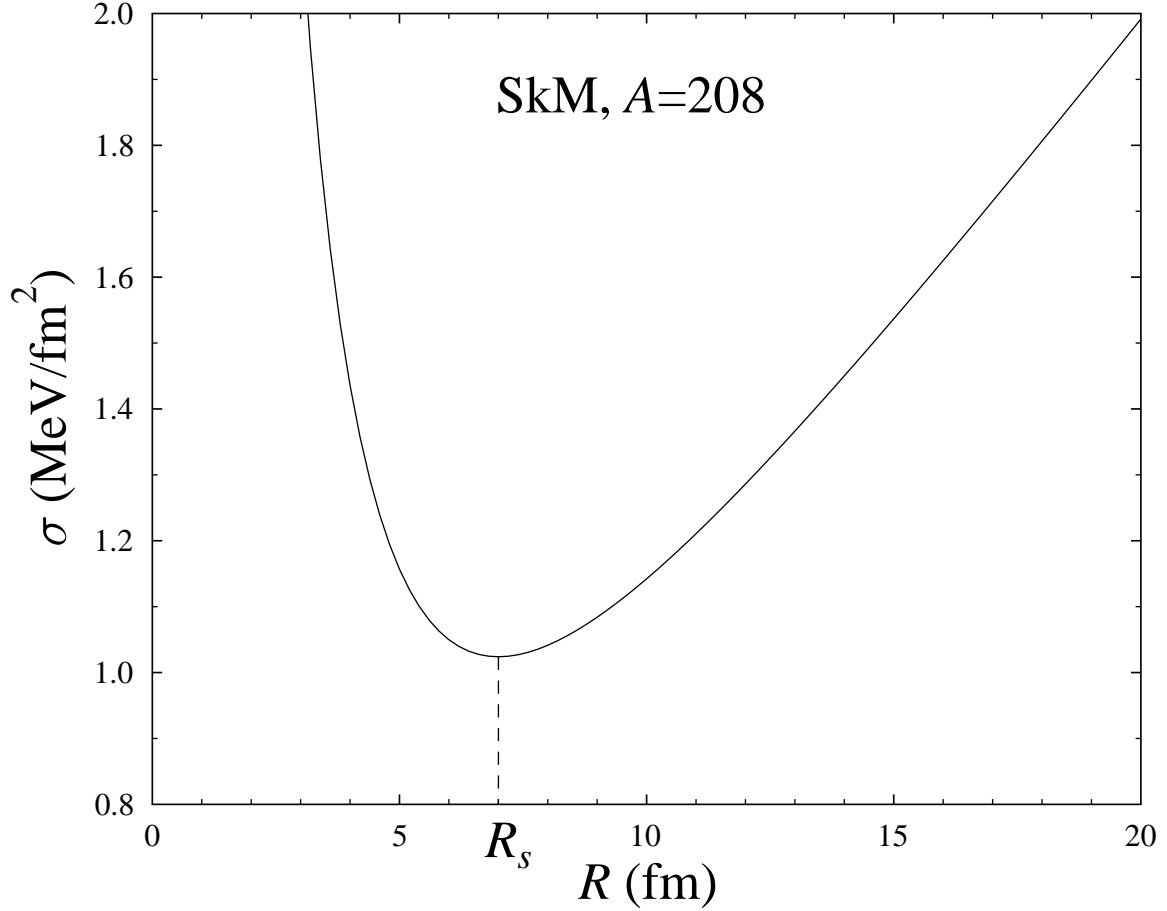


FIG. 1. Surface tension σ as a function of the dividing radius R for $A = 208$. Calculation was performed using *SkM* force. R_s denotes the dividing radius where σ approaches the minimum value.

FIG. 3 illustrates the profile density of the droplet (solid line) and its volume part (dashed line). One can see from this figure that the density of nuclear matter, ϱ_V , slightly differs from that of the droplet bulk, $\rho_{+,0}$. Using *SkM* force for $A = 208$ we obtain slightly different values of $\varrho_V = 0.171 \text{ fm}^{-3}$ and $\rho_{+,0} = 0.170 \text{ fm}^{-3}$. This difference will disappear for incompressible liquid or in the planar limit. In ref. [3] the approximation $\rho_{+,0} = \varrho_V$ was used when obtaining the curvature correction to the surface tension. Since the correction for curvature is calculated in the limit of semi-infinite matter, such approximation will, obviously, give correct results. Note also that both particle densities ϱ_V and $\rho_{+,0}$ exceed the nuclear matter density ρ_∞ (dotted line in FIG. 3). That is because the surface pressure, which influences the bulk properties, leads to an increase in the nucleon density in center of the nucleus.

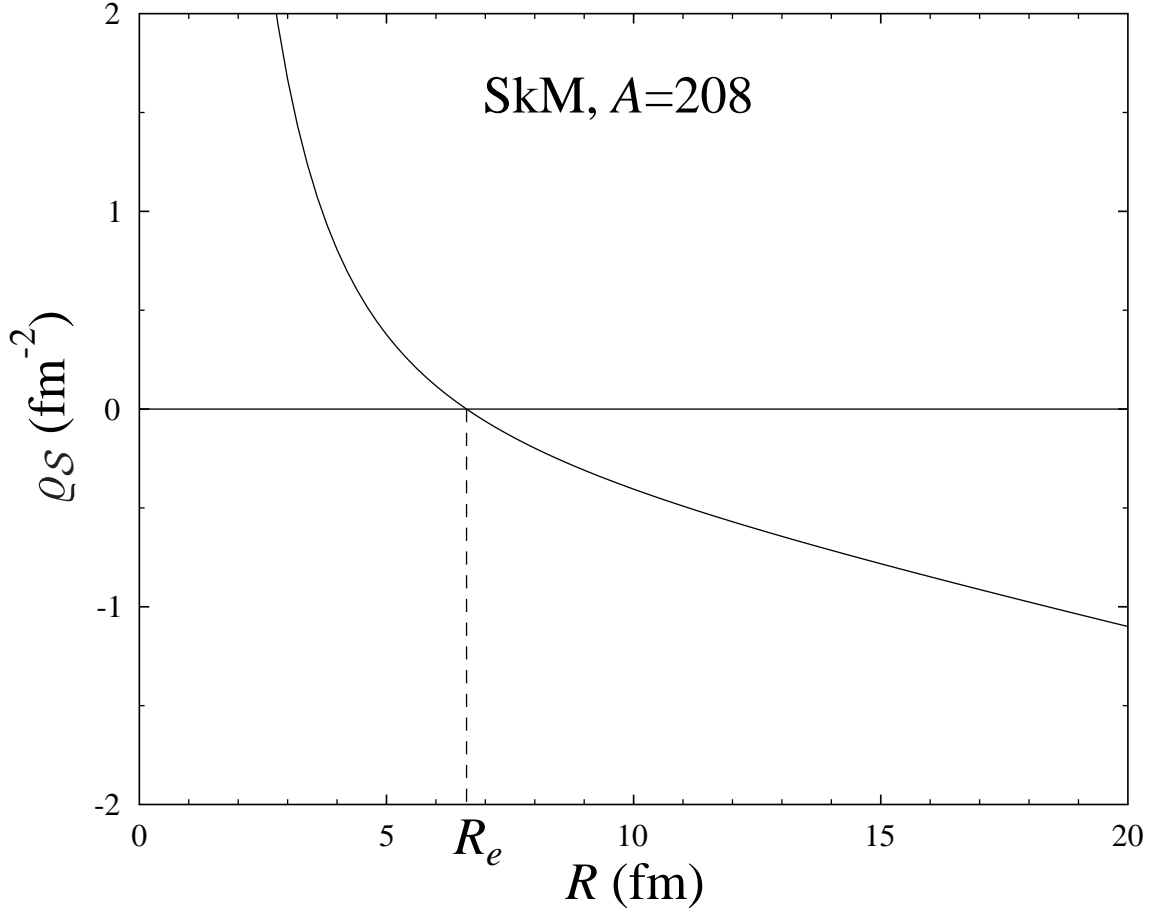


FIG. 2. Surface particle density ϱ_S versus dividing radius R for $A = 208$. Calculation was performed using SkM force. R_e denotes the equimolar radius where ϱ_S becomes zero.

Considering the arbitrary choice of the dividing surface we have determined two radii, the equimolar dividing radius R_e which corresponds to zero surface density ϱ_S and the radius of tension R_s which corresponds to the minimum value of surface tension. From eqs. (7), (10) the values of these radii are given by

$$R_e = \left(\frac{4\pi\varrho_V}{3A} \right)^{-1/3}, \quad R_s = \left(-\frac{2\pi\varrho_V}{3A} \frac{\omega_V}{\omega} \right)^{-1/3}. \quad (16)$$

Below we will assume that the physical (measurable) value of surface tension is that taken at the equimolar dividing surface. Taking eq. (9) for $R = R_s$, using eqs. (15) and (1) and introducing small value $\eta = R_e - R_s$, we obtain

$$P = \frac{2\sigma_\infty}{R_s} \left(1 - \frac{2\xi}{R_s} + \mathcal{O}(R_s^{-2}) \right). \quad (17)$$

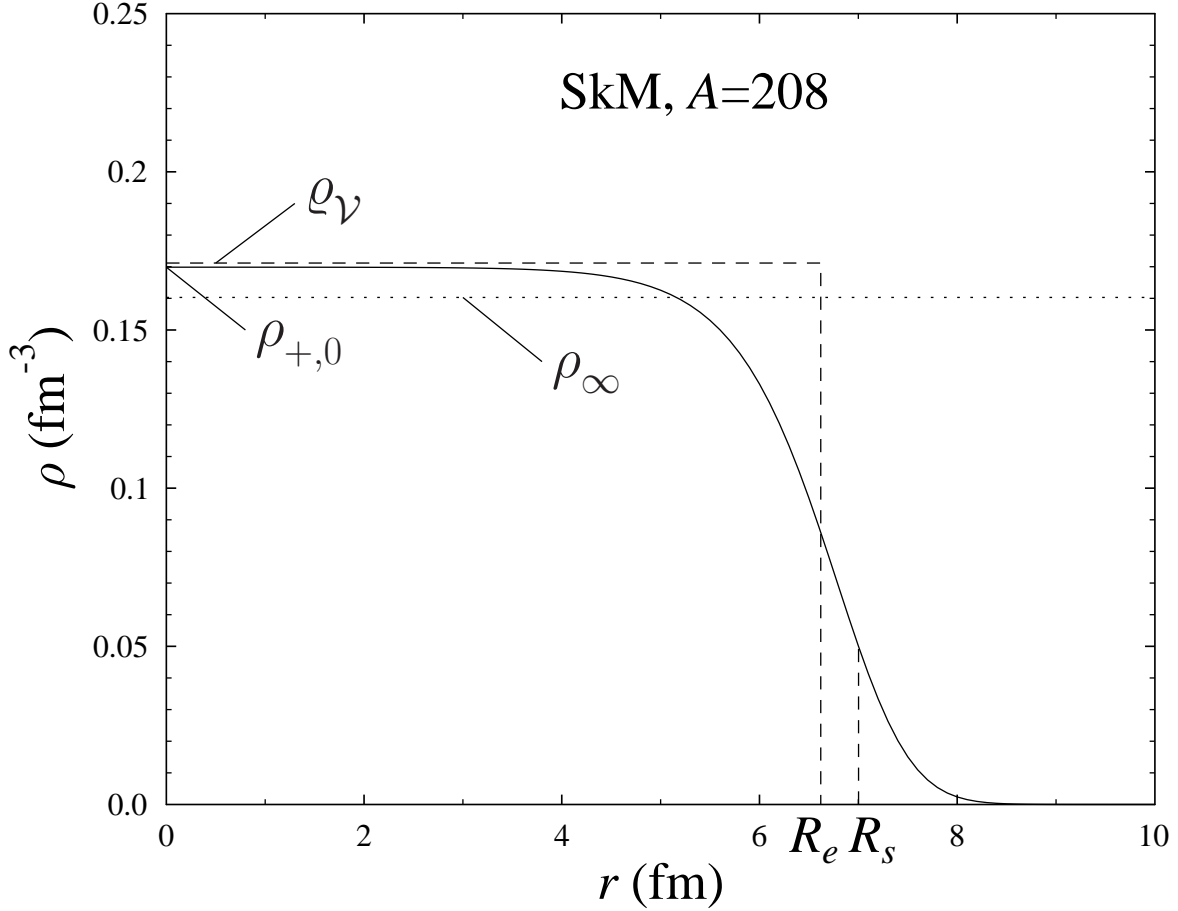


FIG. 3. Profile density $\rho(r)$ for $A = 208$. Solid line shows calculation for the actual droplet, dashed line corresponds to the equimolar distribution dotted line is the particle density ρ_∞ in nuclear matter. Calculation was performed using SkM force. R_e denotes the equimolar radius, R_s is the radius of surface tension.

Taking eq. (9) for $R = R_e$ and eqs. (1) we find

$$P = \frac{2\sigma_\infty}{R_s} \left(1 - \frac{\xi + \eta}{R_s} + \mathcal{O}(R_s^{-2}) \right). \quad (18)$$

We note, that one should make a difference between formal derivative $\sigma'[R]$ in (9) and $\sigma'(R)$ where the surface tension is treated as a function of physical size. However, for the special case of the equimolar dividing surface one can prove that $\sigma'[R_e] = \sigma'(R_e)$, see [10]. In particular, using eq. (1) one finds $\sigma'(R_e) = \sigma_\infty (2\xi R_e^{-2} + \mathcal{O}(R_e^{-3}))$ and, in contrast to $\sigma'[R_s] = 0$, one has $\sigma'(R_s) = \sigma_\infty (2\xi R_s^{-2} + \mathcal{O}(R_s^{-3}))$. Comparing eqs. (17) and (18) for $R_s \rightarrow \infty$, we obtain Tolman result [2] (see also [20])

$$\xi = \lim_{A \rightarrow \infty} (R_e - R_s). \quad (19)$$

This result leads to the important conclusions which were not mentioned in previous studies of nuclear surface. First, one needs to define two different radii: the equimolar radius, R_e , for the proper separation of the surface energy from the total energy of nucleus and determination of the droplet size, and the radius of tension, R_s , to determine the capillary pressure. Second, to obtain the non-zero value of Tolman length, and, consequently, the value of the curvature correction $\Delta\sigma_{\text{curv}} \neq 0$, the droplet must have the finite diffuse surface layer.

The value of Tolman's length could be positive or negative. Positive value of Tolman's length $\xi > 0$ means $\sigma_e < \sigma_\infty$ (see eq. (1)) and negative one gives $\sigma_e > \sigma_\infty$ for curved surface.

IV. NUMERICAL RESULTS

Since we consider a non-charged droplet (without Coulomb), the calculations is possible up to very high values of particle number $A \sim 10^6$. FIG. 4 shows the result of calculation for tension σ_e as a function of doubled droplet curvature $2/R_e$. Calculation was carried out using SkM force. FIG. 4 demonstrates the negative value of ξ for this calculation. An extrapolation of curve in FIG. 4 to zero curvature $2/R_e \rightarrow 0$ allows to derive both the surface tension coefficient $\sigma_\infty = \sigma_e(R_e \rightarrow \infty)$ in a planar geometry and the slope of curve which gives the Tolman length ξ . The result of such kind of extrapolation of $\sigma_e(R_e)$ is shown in FIG. 4 by dashed line.

We have determined the Tolman's length ξ and the planar surface tension σ_∞ for several parametrization of Skyrme interaction. For this purpose we have fulfilled calculations up to particle number 10^6 and extrapolate them to zero curvature. Results are summarized in table I. To obtain the error of the extrapolation $2/R_e \rightarrow 0$ we estimated the magnitude of the higher order term $\sim R_e^{-2}$ in (1). For the interval of particle numbers $A = 10^4 \div 10^6$ we gain the term of about $0.5R_e^{-2}$ for the SkM interaction, so one has here about of $10^{-2}\%$ contribution from this term to the surface tension. One should expect the same accuracy for the extracted values σ_∞ and ξ .

We can see from table I that Tolman's length ξ is negative for nuclear Fermi-liquid drop. This conclusion is also supported by the results of ref. [3]. The value of the Tolman length is only slightly sensitive to the Skyrme force parametrization with the exception of old one SIII.

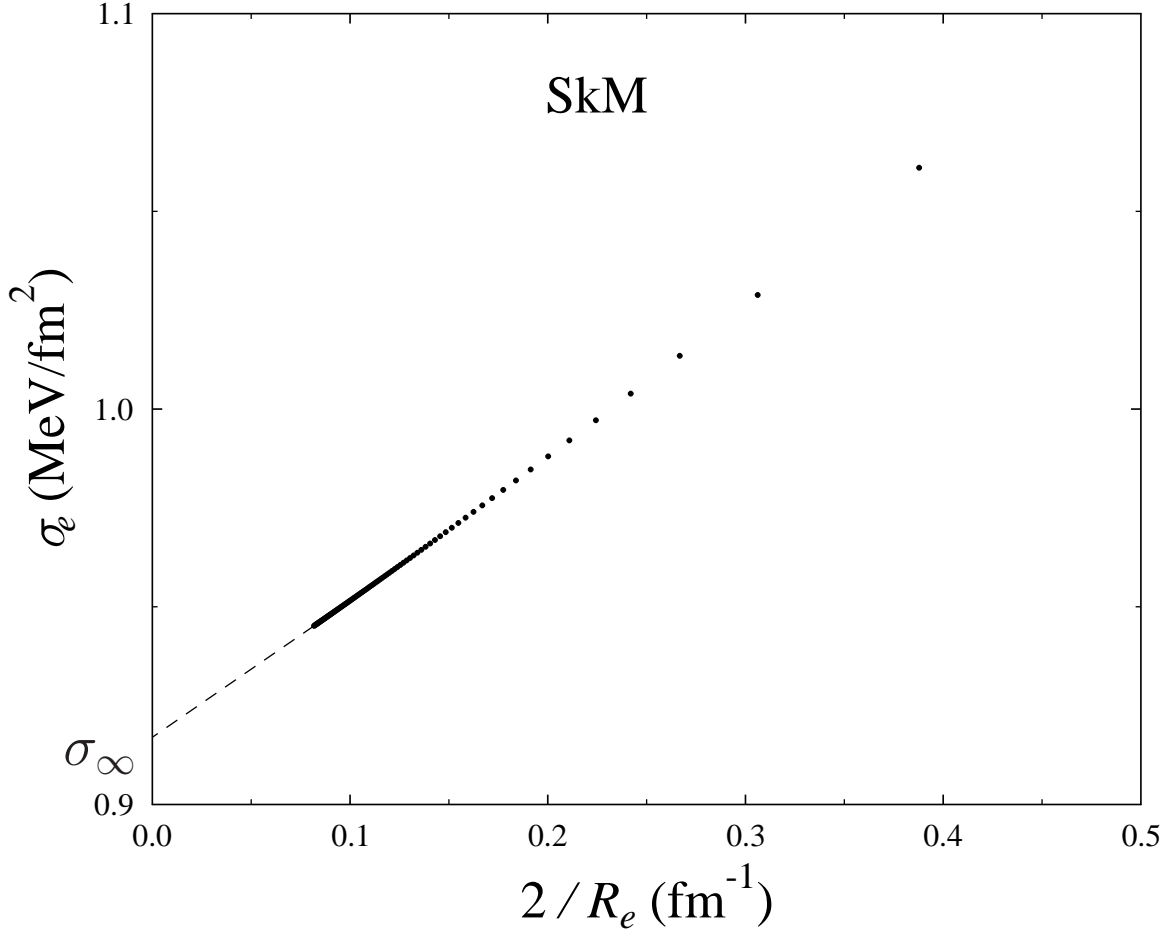


FIG. 4. Surface tension of the droplet versus the surface curvature for the range of particle number $A = 10^2 \div 10^4$. Calculation was performed using SkM force.

The calculation of the curvature correction to the surface tension by use the expansion around the plane surface (semi-infinite nuclear matter) with respect to the surface curvature was introduced in [3] and it was widely used for different types of nucleon-nucleon interactions including the Skyrme-type interactions (see, for example, [15, 22].) Comparing the values the surface, a_2 , and curvature, a_3 , coefficients obtained for T6 and SIII forces in [22] with the analogous results for the same forces of table I by means of eqs. (30), (32) (see the next section), one can see the numerical coincidence of our results with that of [22]. The main reason of this coincidence, in our opinion, is the following. For the Gibbs – Tolman approach at the limit $A \rightarrow \infty$ one has $\varrho_V = \rho_\infty$ and, consequently, the equimolar radius given by eq. (16) and obtained from the condition $\varrho_S = 0$ becomes equal to the equivalent ”sharp” radius of the approach proposed in Ref. [3]. In other words, the applicability of the

TABLE I. Values of Tolman's length ξ and planar surface tension σ_∞ obtained for different parameterizations of Skyrme forces.

Force	ξ (fm)	σ_∞ (MeV/fm ²)
SkM	-0.36	0.92
SIII	-0.26	0.93
SLy230b	-0.37	1.01
T6	-0.36	1.02

Myers – Swiatecky approach is realized in this case. The more detailed comparison is quite difficult since the Gibbs – Tolman approach does not rely on the bulk asymptotics of the energy density functional.

In fact, the above comparison shows the equivalence of two approaches at large masses. It is interesting to analyze the applicability of them for the case of small mass numbers. Following [15, 22] the coefficients of mass formula are calculated using the leptodermous approximation which requires the surface layer thickness to be small as compared to the nuclear size given by the corresponding sharp radius. According to Gibbs [11] the thermodynamical relation (2) remains exact up to the zero value of R_s , provided that the pressure is calculated for the matter at the value of chemical potential of the actual drop. Another conclusion concerning the Gibbs – Tolman approach was made in [10], namely, the low limit of R_s where the definition of the surface tension make sense is about of $R_s \sim |\xi|$. In any case, even though we will require the absolute value of the Tolman length to be small as compared to R_s , the Gibbs – Tolman definition of the surface tension seems more preferable than that obtained using the leptodermous approximation. The reason is that the estimated value for $|\xi|$ (see table I) is lower than the thickness, t , of the surface layer (see [22], table 5).

V. LINK TO THE DROPLET MODEL

The Gibbs concept of dividing surface does not imply any specific energy density functional and relies on the value of energy and the chemical potential which are measurable quantities. It is possible to apply this concept to the droplet model as well. We will consider

non-charged ($N = Z$, without Coulomb interaction) droplet at zero temperature and apply the same procedure as described in previous sections to extract the value of Tolman length. According to [3], one can write free energy, F , and the chemical potential, λ , of the nucleus having mass number A as

$$F = -a_1 A + a_2 A^{2/3} + \left(a_3 - \frac{2a_2^2}{K} \right) A^{1/3} , \quad (20)$$

$$\lambda = -a_1 + \frac{2}{3} a_2 A^{-1/3} + \frac{1}{3} \left(a_3 - \frac{2a_2^2}{K} \right) A^{-2/3} , \quad (21)$$

where a_1 , a_2 and a_3 are, respectively, the volume, the surface and the curvature correction coefficients, K is the incompressibility coefficient. From eqs. (20) and (21) one has the grand potential per particle $\omega = F/A - \lambda$ as

$$\omega = \frac{1}{3} a_2 A^{-1/3} + \frac{2}{3} \left(a_3 - \frac{2a_2^2}{K} \right) A^{-2/3} \quad (22)$$

The equation of state for infinite nuclear matter in terms of droplet model reads

$$e = -a_1 + \frac{1}{2} K \epsilon^2 \quad (23)$$

for the free energy per particle, e , and

$$l = -a_1 + \frac{1}{6} K \epsilon (9\epsilon - 2) \quad (24)$$

for the chemical potential, l , beyond the equilibrium point. In eqs. (23) and (24), the dimensionless variable

$$\epsilon = -\frac{1}{3} \frac{\rho - \rho_\infty}{\rho_\infty} \quad (25)$$

was introduced as the measure of difference between nuclear matter density ρ and its equilibrium value ρ_∞ . Fixing the value of particle density $\varrho_V = \rho(\lambda)$ from the condition $l(\rho) = \lambda$, one obtains the volume part of grand potential per particle

$$\omega_V = -\frac{2}{3} a_2 A^{-1/3} - \frac{1}{3} \left(a_3 - \frac{8a_2^2}{K} \right) A^{-2/3} + \mathcal{O}(A^{-1}) , \quad (26)$$

and, using also (22), the ratio ω/ω_V

$$\frac{\omega}{\omega_V} = -\frac{1}{2} - \frac{3}{4} \frac{a_3}{a_2} A^{-1/3} + \mathcal{O}(A^{-2/3}) . \quad (27)$$

As seen from eq. (27) the compression effect is canceled out from ω/ω_V up to the order of $A^{-1/3}$. Using eqs. (16) and (19), one derives both radii

$$R_e = r_0 A^{1/3} \left[1 - \frac{2a_2}{K} A^{-1/3} + \mathcal{O}(A^{-2/3}) \right] , \quad (28)$$

$$R_s = r_0 A^{1/3} \left[1 + \left(\frac{a_3}{2a_2} - \frac{2a_2}{K} \right) A^{-1/3} + \mathcal{O}(A^{-2/3}) \right] \quad (29)$$

and the Tolman length

$$\xi = -\frac{a_3}{2a_2} r_0 \quad (30)$$

where $r_0 = (4\pi\rho_\infty/3)^{-1/3}$. Taking the eq. (6) at $R = R_e$, by the use of the eqs. (22), (26) and (28), the surface tension reads

$$\sigma_e = \frac{(\omega - \omega_V)A}{4\pi R_e^2} = \frac{1}{4\pi r_0^2} (a_2 + a_3 A^{-1/3} + \mathcal{O}(A^{-2/3})) . \quad (31)$$

With Tolman length given by (30) and relation

$$\sigma_\infty = \frac{a_2}{4\pi r_0^2} \quad (32)$$

one can reduce eq. (1) to (31). As seen from the above eqs. (28), (29) and (30), both the equimolar, R_e , and the tension, R_s , radii include the term of compression effect $(2a_2/K)A^0$, whereas the value of Tolman's length ξ of eq. (19) reflects purely the effect of curvature of dividing surface. Using the results presented in table I, one may estimate the ratio of the curvature correction to the surface coefficient of droplet model as $a_3/a_2 \approx 0.63$ for the case of SkM nucleon-nucleon interaction. This value of the ratio a_3/a_2 is consistent with that of [4].

VI. CONCLUSIONS

Considering a small droplet with a finite diffuse layer, we have introduced a formal dividing surface of radius R which splits the droplet onto volume and surface parts. The corresponding splitting was also done for the free energy. Assuming that the dividing surface is located close to the interface, we are then able to derive the volume pressure P and the surface free energy F_S . In general, the surface free energy F_S includes the contributions from the surface tension σ and from the binding energy of A_S particles within the surface layer. The equimolar surface and the actual physical size of the droplet was derived by the condition $\varrho_S = 0$.

In a small nucleus, the diffuse layer and the curved interface affect the surface properties significantly. In agreement with Gibbs-Tolman concept [2, 11], two different radii have to be introduced in this case. The first radius, R_s , is the surface tension radius which provides the

minimum of the surface tension coefficient σ and the satisfaction of the Laplace relation (15) for capillary pressure. The another one, R_e , is the equimolar radius which corresponds to the equimolar dividing surface and defines the physical size of the sharp surface droplet, i.e., the surface at which the surface tension is applied. The difference of both radii $R_e - R_s$ in an asymptotic limit of large system $A \rightarrow \infty$ derives the Tolman length ξ . That means that the presence of curved surface is not sufficient for the calculation of the curvature correction to the surface tension. The finite layer in the particle distribution is required. We point out that the Gibbs-Tolman theory allows to treat a liquid drop within thermodynamics with minimum assumptions. Once the binding energy and chemical potential of the nucleus are known its equimolar radius, surface tension radius and surface energy can be evaluated by use of equation of state of the infinite nuclear matter. In this sense, in contrast to the "geometrical" definition of nuclear size [8], the Gibbs-Tolman approach does not rely on details of the particle density profile. In particular, the quantum oscillations of bulk density [13] do not need to be smoothed to obtain the volume density ϱ_V which is different, in general, than the bulk density.

The sign and the magnitude of the Tolman length ξ depend on the interparticle interaction. We have shown that the Tolman length is negative for the nuclear Fermi liquid drop. As a consequence of this the curvature correction to the surface tension could lead to the hindrance of the yield of light fragments at the nuclear multifragmentation in heavy ion collisions.

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